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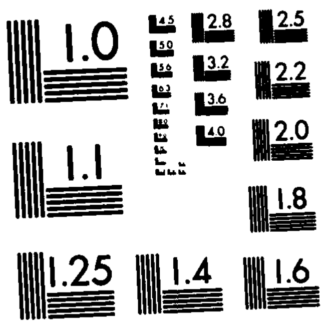

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INVALIDITY OF LOCAL THERMODYNAMIC EQUILIBRIUM FOR ELECTRONS IN THE SOLAR TRANSITION REGION. II. ANALYSIS OF A LINEAR BGK MODEL

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Abstract

In an earlier paper ~~(Shoub 1983)~~ numerical solutions of the Landau equation were obtained which show that the tail of the electron velocity distribution function differs substantially from a local Maxwellian distribution in the solar transition region and upper chromosphere. In this paper, I show that a linearized version of the BGK model kinetic equation, with collision frequency proportional to v^{-3} (1/v cubed), can be solved analytically for the tail of the distribution function in an atmosphere with prescribed temperature and density profiles. Results for the angle-averaged distribution so obtained are shown to be in reasonably good agreement with earlier numerical results. Accurate, easily evaluated approximations for the tail of the distribution function are derived from the exact formulas. These show that both the zeroth and first angular moments of the distribution function are nearly power laws over a wide velocity range in the low transition region. I also show that the heat flux into lower temperature region is carried by suprathermal electrons with velocities well above local thermal velocities. The formulas given here should be useful in the calculation of electron-ion inelastic collision rates under conditions in which the local Maxwellian approximation is invalid.

A

I. Introduction

In Shoub (1983; hereafter referred to as Paper I) numerical solutions of the Landau equation were obtained for the electron velocity distribution function (EVDF) in idealized models of the solar transition region. These results showed that the EVDF forms a pronounced, anisotropic, high-velocity tail in the low transition region ($T \leq 3 \times 10^5$ K) and upper chromosphere as the result of the streaming of fast electrons down the temperature gradient. It was also shown that, in consequence, inelastic electron-ion collision rates are signficantly enhanced in these regions relative to their values obtained using a local-Maxwellian energy spectrum.

The results reported in Paper I were obtained through involved numerical calculations whose nature and expense hinder the development of physical intuition concerning the underlying transport processes. In this complementary paper I therefore address the same problem as considered in Paper I, but here use a Bhatnagar-Gross-Krook (1954) model for the collision term. Moreover, I take advantage of my earlier finding that only the tail of the EVDF differs significantly from a local Maxwellian under transition region conditions in order to linearize the BGK collision term and thereby obtain an analytically soluble kinetic equation. In this approximation one can obtain an expression for the EVDF in an atmosphere with a prescribed temperature and density profile. Results for the angle-averaged distribution function so obtained are shown to be in reasonably good agreement with those found earlier using the Landau equation.

Although the analytical solutions I obtain are relatively complicated [see equations (7a) and (7b)], they can be accurately approximated at supra-thermal velocities by much simpler expressions which provide insight into the underlying physical processes. These formulas will hopefully be useful to workers wishing to evaluate electron-ion inelastic collision rates under conditions in which electrons do not have a local Maxwellian energy spectrum.

Two new results which emerge from the present work are a) that the angle-averaged distribution function is nearly a power law over a wide velocity range in the lower temperature regions of the atmosphere, and b) that the conductive energy flux into the lower temperature regions is carried predominantly by suprathermal electrons with velocities 3 to 25 times local thermal velocities. The latter result shows clearly that classical transport theory, which predicts that the heat flux is carried by electrons with velocities two to three times local thermal velocities, is invalid under conditions met in the solar transition region.

II. Formulation

Consider an inhomogeneous but isobaric slab of fully ionized hydrogen in which the protons are idealized as being infinitely massive and at rest. The slab has thickness L, and any magnetic field present is assumed to be parallel to the gradient direction, which I take to be the z-axis. Let $f(\mu, v, z)$ denote the EVDF, with $\underline{v}$ the electron velocity, and $\mu = \underline{v}\cdot\hat{e}_z/|\underline{v}|$. The form of the BGK equation I shall use is then

$$\mu v \frac{df}{dz} = \nu(v,z)\,\{f^*(\mu,v,z) - f(\mu,v,z)\} \quad , \qquad (1a)$$

where

$$f^*(\mu,v,z) = n'(z)(m/2\pi kT'(z))^{3/2}\exp[-m(\underline{v}-w'\hat{e}_z)^2/2kT'(z)] \qquad (1b)$$

is a local-Maxwellian distribution with parameters n', w', and T' and (cf., Rawls, Chu, and Hinton 1975), and

$$\nu(v,z) = \Gamma\cdot\frac{16\pi e^4 n\ln\Lambda}{m^2 v^3}\,\{\mathrm{erf}(\xi) - \frac{2}{\sqrt{\pi}}\,\xi\exp(-\xi^2)\} \qquad (1c)$$

with $\xi = v/v_{th}$ and $v_{th}^2 = 2kT/m$. Here m is the electron mass, $\ln\Lambda$ is the coulomb logarithm, and Γ is a constant to be chosen later. When $\Gamma = 1$, ν^{-1} is the slowing down time of a test electron in a fully ionized hydrogen plasma (Krall and Trivelpiece 1973).

Several remarks concerning equation (1a) are in order. First, note that although at first sight (1a) appears to be a linear equation, it is not. In order to conserve particles, momentum, and energy, the parameters n', T' and w' appearing in f^* must be chosen to satisfy the constraints particles

$$\int_{-1}^{1} d\mu \int_{0}^{\infty} dv\ v^2\ \nu(v)[f^* - f] = 0 \tag{2a}$$

momentum

$$\int_{-1}^{1} \mu\ d\mu \int_{0}^{\infty} dv\ v^3\ \nu(v)[f^* - f] = \frac{1}{2} \int_{-1}^{1} \mu d\mu \int_{0}^{\infty} dv\ v^3\ \nu_{ei}(v)[\partial_\mu\ \{(1-\mu^2)\partial_\mu f\}]$$

$$= - \int_{-1}^{1} \mu d\mu \int_{0}^{\infty} dv\ v^3\ \nu_{ei}(v)\ f(\mu\ ,v) \tag{2b}$$

energy

$$\int_{-1}^{1} d\mu \int_{0}^{\infty} dv\ v^4\ \nu(v)[f^* - f] = 0 \tag{2c}$$

and are, therefore, functionals of f. Here $\nu_{ei} = 4\pi\ e^4 n \ln\Lambda/(m^2 v^3)$ is the electron-proton momentum transfer collision frequency, and $1/2\ \nu_{ei}(v)\ \partial_\mu\ [(1 - \mu^2)\partial_\mu f]$ is the electron-proton collision term appearing in the Landau equation (to leading order in m/m_p). Constraint (2b) thus forces our model equation to give the correct electron-proton momentum transfer. In general, therefore, the BGK equation is highly nonlinear and consequently is analytically intractable. Fortunately, our present interest in conditions for which the bulk of the electrons have nearly a locally Maxwellian distribution (see Paper I) permits reasonable estimates of n', T', and w' to be guessed _a priori_. For inspection of equations (2a-2c) shows that the condition f nearly Maxwellian at thermal velocities implies that n', T' and w' will be close to the corresponding thermodynamic variables which, in turn, may be estimated using macroscopic (i.e., classical) arguments. I therefore choose

$$T'(z)^{7/2} = T_c^{7/2} + (T_h^{7/2} - T_c^{7/2})\, z/L \quad , \tag{3a}$$

$$n'T' = \text{constant} \quad , \tag{3b}$$

$$w' = 0 \quad , \tag{3c}$$

corresponding to an isobaric slab of thickness L in which energy is transferred solely by (classical) thermal conduction. [To arrive at (3a) I have treated $\ln\Lambda$ as a constant.] Here T_h and T_c are temperatures characterizing the incoming EVDF at the upper and lower boundaries, respectively. With n', T' and w' regarded as known functions of position, equation (1a) reduces to a linear, ordinary differential equation in which μ and v appear as parameters. This represents a significant mathematical simplification relative to the Landau equation, where μ and v enter as independent variables. Note, however, that because the above constraints are violated in this approximation, one cannot obtain a kinetic-theoretic prediction of the electron temperature profile. One must solve the nonlinear problem for this.

In order to retain the advantage mentioned above, I have omitted a thermal electric field term from the lefthand side of (1a). This is not a serious omission for my present purposes because, as argued in detail in Paper I, this term is unimportant relative to the streaming term at suprathermal velocities.

It is useful to note that by analogy with the equation describing photon transport in a medium in which the matter is in a state of local thermodynamic equilibrium, the linearized BGK equation can be interpreted in the following simple way. Electrons travel linear trajectories at constant velocity, with the pathlength of a given flight being a random variable whose probability distribution is fixed by the electron's speed and collision frequency with background electrons and protons. A given flight is terminated in a single thermalizing event in which the electron is "absorbed" and then "re-emitted" with a new random velocity whose probability distribution is f^*. This, of course, is not how electrons behave. Nevertheless, proper choice of the collision frequency ν allows many aspects of more complicated descriptions (e.g., via the Landau equation) to be modeled accurately.

Since my interest lies in the high-velocity tail of the distribution, one final simplification I shall invoke is to use the large-velocity form of the collision frequency given in equation (1b). It is not necessary to do this, but the resulting formulas are more transparent if I do. Further, I shall omit the primes from n', T' and w', because I no longer distinguish between these variables and their thermodynamic counterparts.

III. Analysis

a) Formal Solution

Let us begin by introducing $x = \ln T$ as the independent spatial variable and use (1c) and (2b) to rewrite (1a) in the form

$$\mu\alpha\xi^4 \frac{df}{dx} = f^* - f \quad , \tag{4a}$$

where

$$\alpha(T) \equiv \lambda(T) \frac{d\ln T}{dz} \quad ; \lambda(T) = (kT)^2/4\pi e^4 n \ln\Lambda \quad . \tag{4b}$$

$$\xi(T) = v/v_{th}(T) \; ; \; v_{th}^2 = 2kT/m \quad . \tag{4c}$$

It follows from (3a), (3b) and (4b) that

$$\alpha(T) = \frac{2}{7} \frac{\lambda(T_h)}{L} \left(\frac{T_h}{T}\right)^{1/2} \left[1 - \left(\frac{T_c}{T_h}\right)^{7/2}\right] . \tag{4d}$$

Equation (4a) is to be solved subject to the boundary conditions

$$f(z=0) = f^*(v;T_c,n_c) = n_c \; (m/2\pi kT_c)^{3/2} \exp[-(mv^2/2kT_c)]; \quad \geq 0 \tag{5a}$$

$$f(z=L) = f^*(v;T_h,n_h) = n_h \; (m/2\pi kT_h)^{3/2} \exp[-(mv^2/2kT_h)]; \quad \leq 0 \tag{5b}$$

with $n_h T_h = n_c T_c$. These are the same boundary conditions as used in Paper I. The choices $T_h = 2 \times 10^6$ K, $T_c = 8.1 \times 10^3$ K, $n_h = 3 \times 10^8$ cm^{-3} and $L = 5 \times 10^9$ cm will hereafter be referred to as the standard model.

The solution of equation (4a) subject to conditions (5a,b) may be written as

$$f(\mu,v;T) = f^*(T_c)\exp[-p_v(T,T_c)/\mu] + \int_{T_c}^{T} f^*(T') \frac{\exp[-p_v(T,T')/\mu]}{\mu\alpha(T')\xi(T')^4}\frac{dT'}{T'};\ 0\leq\mu\leq 1 \ , \quad (6a)$$

and

$$f(\mu',v,T) = f^*(T_h)\exp[-p_v(T,T_h)/\mu'] + \int_{T}^{T_h} f^*(T') \frac{\exp[-p_v(T,T')/\mu']}{\mu'\alpha(T')\xi(T')^4}\frac{dT'}{T'};0\leq\mu'\leq 1 \quad (6b)$$

where $\mu' = -\mu$ and

$$p_v(T,T') = \frac{2}{5\alpha(T')\xi(T')^4}\left[\left|1-\left(\frac{T}{T'}\right)^{5/2}\right|\right] \quad . \quad (6c)$$

The quantity $\exp[-p_v(T,T')/|\mu|]$ is the probability that a (μ,v) electron "emitted" at T' will survive a flight to T. Note that for fixed v, p_v is symmetric in T and T', since $\alpha\xi^4$ is proportional to $T^{-5/2}$. An electron of given velocity is thus equally likely to go from T to T' as from T' to T. However, it is much more likely that an electron "emitted" with a given value of ξ will survive a flight to lower temperatures than one to higher temperatures. This is an important point, because at any given location electrons are available only over a limited range of ξ.

It will be helpful later on to notice that equation (4a) implies that the quantity $\alpha\xi^4$ may be interpreted as the electron mean free path, measured in units of lnT.

Equations (6a,b) may readily be integrated over μ to obtain the zero'th and first angular moments. When written in dimensionless form, the results are:

$$\phi^0(\xi,T) = 1/\sqrt{\pi}\{(T/T_h)^{5/2} \exp(-\xi^2 T/T_h) E_2[p_v(T,T_h)]$$

$$+ (T/T_c)^{5/2} \exp(-\xi^2 T/T_c) E_2[p_v(T,T_c)] \tag{7a}$$

$$+ (\alpha\xi^4)^{-1} \int_{T_c}^{T} \exp(-\xi^2 T/T') E_1[p_v(T,T')] dT'/T'\} \quad ,$$

and

$$\phi^1(\xi,T) = \frac{3}{2}\frac{1}{\sqrt{\pi}} \{(T/T_c)^{5/2} \exp(-\xi^2 T/T_c) E_3[p_v(T,T_c)]$$

$$- (T/T_h)^{5/2} \exp(-\xi^2 T/T_h) E_3[p_v(T,T_h) \tag{7b}$$

$$+ (\alpha\xi^4)^{-1} \int_{T_c}^{T} \exp(-\xi^2 T/T') E_2[p_v(T,T')] dT'/T'$$

$$- (\alpha\xi^4)^{-1} \int_{T}^{T_h} \exp(-\xi^2 T/T') E_2[p_v(T,T')] dT'/T'\} \quad .$$

Here $\phi = (2\pi v_{th}^3/n) f$, $\phi^0 = 1/2 \int_{-1}^{1} \phi \, d\mu$, $\phi^1 = 3/2 \int_{-1}^{1} \mu\phi \, d\mu$, and

$E_n(x) = \int_1^{\infty} t^{-n} \exp(-xt)dt$, $n = 1,2,3,\ldots$, are exponential integral functions.

The right side of equation (7a), evaluated with $r = 1$ and parameter values equal to those of the standard model, is compared to corresponding Fokker-Planck (FP) results in Figure 1. It is seen that although BGK theory (with $r = 1$) predicts somewhat larger deviations from a local Maxwellian than does FP theory, the results are qualitatively similar, at least over the velocity range in which the FP results have been calculated. I find that choosing r = 2.2 gives quite good agreement in the tail of the distribution. Note that since Figure 1 is a log-log plot, the nearly linear behavior of curves at intermediate velocities implies that ϕ^0 is approximately a power law over this velocity range. This result is derived below.

Figure 2 shows the dimensional, angle-averaged distribution $f^o = (2\pi v_{th}^3)^{-1} n\phi^0$ at several locations with corresponding local-Maxwellians shown as dashed lines for comparison. The relatively weak spatial (temperature) dependence of the high-velocity tails should be noted.

A Chapman-Enskog analysis of equation (4a) yields the result $f(\mu,v,T) = f^* + f^1_{cl} + O(\alpha^2)$, where

$$f^1_{cl} = -\alpha\xi^4 \, df^*/dx = \alpha\xi^4(\tfrac{5}{2} - \xi^2)f^* \quad . \tag{8a}$$

Thus, the classical BGK heat flux is

$$q_{cl} \simeq 1/2 \; m \; (2 \;) \; 2/3 \int_0^\infty v^5 \; f^1_{cl} \; dv \tag{8b}$$

$$= 4/3 \; \pi^{-1/2} \; (nkT)v_{th} \; \alpha(T) \int_0^\infty \xi^9(\tfrac{5}{2} - \xi^2) \; \exp(-\xi^2) \; d\xi \; . \tag{8c}$$

The classical and exact heat-flux integrand [as obtained from equation (7b)] are compared at several locations in the standard model in Figure 3. From these results it is clear that the conductive energy flux into the lower temperature regions is carried predominantly by suprathermal electrons.

The heat flux obtained from (7b) and normalized to its classical value is shown as a function of temperature in Figure 4. Results for several pressures are shown; all other parameters have their standard values. Since the temperature profile I have assumed is derived from the condition that the classical flux be constant, the spatial variation of the ratio shown in

Figure 4 is due entirely to the variation of the "actual" heat flux. That this "actual" heat flux exhibits a spatial variation is of course incorrect, and is due to violation of energy conservation (equation (2c)) in the present linear theory. That the calculated flux is not constant implies that the correct temperature profile should be different from the $T^{7/2}$ law assumed. In fact, the spatial variation of the flux shown in Figure 4 implies that the gas is cooled at high temperatures and is heated at low temperatures. Thus, the correct temperature profile will be steeper than the classical profile at high temperatures and less steep at low temperatures. Unfortunately, determination of the correct "constant-flux" profile requires solution of the nonlinear problem and for this reason is deferred to a later publication.

b. Approximate Solutions

The expressions for the distribution function given above (i.e., equations (6a,b) and (7a,b)) are difficult to evaluate and to interpret. In this section I derive useful approximations to these expressions.

Consider the integral in equation (6b). Using (2b), (3a,c) and (4c,d), it can be rewritten in the form

$$J(\mu',v,T) = C_M(T)(\mu'\alpha\xi^4)^{-1}\int_T^{T_h} \exp[-\xi^2\, h(T')]\, \frac{dT'}{T'} \quad , \tag{9a}$$

where $\mu' = -\mu$, $C_M(T) = n(T)(m/2\,\pi kT)^{-3/2}$ (Note: $C_M(T) \propto T^{-5/2}$) , and

$$h(T') = \frac{T}{T'} + \frac{2}{5}\left(\frac{T}{T^*}\right)^{7/2}\left[\left(\frac{T'}{T^*}\right)^{5/2} - 1\right]. \tag{9b}$$

Here, as elsewhere, temperature-dependent quantities written without arguments are to be evaluated at T. The quantity $h(T')$ has a minimum at $T' = T^*$, where

$$\left(\frac{T^*}{T}\right)^{7/2} = \mu'\alpha\xi^6 \quad . \tag{10}$$

Note that since both $\alpha(T)$ and $\xi(T)$ are proportional to $T^{-1/2}$, it follows from (10) that T^* is independent of T; i.e., $T^* = T^*(\mu',v)$. This point will prove significant later on. For large ξ^2 the integrand in (9a) is sharply peaked about $T' = T^*$; the integral may therefore be accurately approximated by expanding $h(T')$ in a Taylor series about $T' = T^*$, i.e., $h(T') \simeq h(T^*) + 1/2h''(T^*)\,(T' - T^*)^2$, provided, of course, that $T \leq T^* \leq T_h$. Substituting this expansion into (9a) and making use of the relations

$$h(T^*) = \frac{7}{5}\left(\frac{T}{T^*}\right)\left[1 - \left(\frac{T}{T^*}\right)^{5/2}\right] \quad , \tag{11a}$$

$$h''(T^*) = \frac{7}{2}\left(\frac{1}{T^{*2}}\right)\left(\frac{T}{T^*}\right) \tag{11b}$$

gives

$$J(\mu',\xi,T) = \frac{C_M(T)}{\mu'\alpha\xi^4} \exp\left[-\xi^2 h(T^*)\right] \int_{-\infty}^{\infty} \frac{T^*}{T'} \exp\left[-\frac{7}{4}\xi(T^*)^2 \left(\frac{T'-T}{T^*}\right)^2\right] d\left(\frac{T'-T^*}{T^*}\right) \tag{12a}$$

$$= \left[\frac{4}{7\xi(T^*)^2}\right]^{1/2} \frac{C_M(T)}{\mu'\alpha\xi^4} \exp\left[-\xi^2 h(T^*)\right] \tag{12b}$$

$$= f^*(T^*)\left[\frac{4\pi}{7\xi(T^*)^2}\right]^{1/2} \frac{1}{\mu'\alpha(T^*)\xi(T^*)^4} \cdot \exp\left[-\frac{2}{5}\xi(T^*)^2\left[1 - \left(\frac{T}{T^*}\right)^{5/2}\right]\right] \tag{12c}$$

where

$$\xi(T^*) = \left(\frac{T}{T^*}\right)^{1/2} \xi(T) = \left(\frac{\xi}{\mu'\alpha}\right)^{1/7} \quad , \tag{12d}$$

and

$$f^*(T^*) = n(T^*)\ (m/2\pi kT^*)^{-3/2}\ \exp[-\xi(T^*)^2] \quad . \tag{12e}$$

In going from (12b) to (12c) I have used the fact that $C_M(T) \propto T^{-5/2}$, and that $\mu' \alpha(T^*) \xi(T^*)^6 = 1$, which follows from (10). Equation (12d) is valid provided $T < T^* < T_h$ or, equivalently, provided $1 < \alpha\xi^6 < (T_h/T)^{7/2}$ and $\mu' > (\alpha\xi^6)^{-1}$.

It is easily shown that when $T^* < T_h$ the integral term calculated above dominates the upper boundary term in equation (6b). Thus,

$$f(\mu',v,T) = \left[\frac{4\pi}{7\xi(T^*)^2}\right]^{1/2} (\mu' \alpha\xi^4)^{-1}_{T=T^*}\ f^*(T^*) \exp\left[-\frac{2}{5}\xi(T^*)^2\left[1 - \left(\frac{T}{T^*}\right)^{\frac{5}{2}}\right]\right] \tag{13a}$$

which is valid provided

$$1 < \alpha\xi^6 < \left(\frac{T_h}{T}\right)^{7/2} \qquad \text{and} \qquad (\alpha\xi^6)^{-1} < \mu' < 1 \quad . \tag{13b}$$

where $T^*(\mu',v)$ is defined in equation (10).

Equation (13a) is readily interpreted: the distribution function at μ',v and T is equal to a local Maxwellian distribution evaluated at v and at a higher temperature $T^*(\mu',v)$, times the probability $\exp[-\frac{2}{5}\xi(T^*)^2\ \{1 - (\frac{T}{T^*})^{5/2}\}]$ that a (μ',v) electron "emitted" at T^* will survive a flight to T, times the interval $\left[\frac{4\pi}{7\xi(T^*)^2}\right]^{1/2}$ in lnT about $\ln T^*$, over which (μ',v) electrons are effectively emitted. Note that since T^* is independent of T, f acquires a dependence on T only through the $1 - (\frac{T}{T^*})^{5/2}$ factor appearing in the argument of the exponential. For large ξ this dependence is quite weak, as may be seen from Table I, where values of $\frac{T^*}{T}$ and $\xi(T^*)$ are listed for several values of their arguments.

Since T^* is the temperature at which a suprathermal (μ,v) electron $(\mu<0)$ suffered its last thermalizing collision, it is seen from Table I that $f(\mu,v,T)$ becomes increasingly nonlocal in character as ξ increases and as $\mu\to-1$. Note also that the quantity $\xi(T^*)$ has several meanings: i) it is the value of ξ with which (μ,v) electrons leave T^*; from Table I is it seen that $2.25 < \xi(T^*) < 3.25$. Thus it is the near-thermal part of the distribution which contributes electrons to lower temperature regions. ii) The quantity $\left[\frac{4\pi}{7\xi(T^*)^2}\right]^{1/2} \simeq 1.34/\xi(T^*)$ is the range of $\ln T$, centered on $\ln T^*$, which contributes (μ,v) electrons to lower temperatures. iii) The quantity $\exp\left[-\frac{2}{5}\xi(T^*)^2\ \{1-(\frac{T}{T^*})^{5/2}\}\right]$ is the probability that a (μ,v) electron "emitted" near T^* will survive a flight to T. Surprisingly perhaps, this probability decreases as ξ increases (because T^* increases).

An approximate expression for the distribution function which is valid for $T^* > T_h$ (i.e., for $\alpha\xi^6 > \left(\frac{T_h}{T}\right)^{7/2}$ and $\mu< 0$) is easily obtained from (9a) by expanding the quantity $h(T')$ in a Taylor series about $T' = T_h$. Adding the upper boundary term to the result I find that

$$f(\mu,\xi,T) = C_M(T)\ \{\exp[-\xi^2\ h(T_h)] - \left(\frac{T_h}{T^*}\right)^{7/2} \exp[-\xi^2\ h(T)]\}[1 + 0\left(\frac{T_h}{T^*}\right)^{7/2}]$$

$$\alpha\xi^6 > \left(\frac{T_h}{T}\right)^{7/2} \quad ; -1 \leq \mu <- (\alpha\xi^6)^{-1} \tag{14a}$$

where, from (9b), $h(T) = 1$ and

$$h(T_h) = \frac{T}{T_h}\left[1 + \frac{2}{5}\left(\frac{T_h}{T^*}\right)\ \left[\left(\frac{T_h}{T^*}\right)^{5/2} - \left(\frac{T}{T^*}\right)^{5/2}\right]\right] \leq 1 \quad . \tag{14b}$$

The inequality in (14b) follows from the conditions $T < T_h$ and $T^* > T_h$. Note that as ξ increases, $\frac{T_h}{T^*}$ and $\frac{T}{T^*}$ decrease and, in consequence, f tends to

$f^*(T_h)$. Note also that $f(T_h) = f^*(T_h)$, as required by the boundary condition (5b).

An approximate expression for the distribution of upward ($\mu > 0$) propagating suprathermal electrons can be obtained by expanding the argument of the exponential in the integrand (in 6a) about $T' = T$. The result may be written

$$f(\mu, \xi, T) = \frac{f^*(T)}{1 + (\frac{T^*}{T})^{7/2}} + C_M(T_c) \exp[-\xi^2 q(T_c)] \left[1 - \frac{(T/T_c)^{5/2}}{1 + (\frac{T^*}{T})^{7/2}}\right] ; \quad (15a)$$

$$0 < \mu \leq 1 \quad ,$$

where

$$q(T') = \frac{T}{T'} + \frac{2}{5}\left(\frac{T}{T^*}\right)^{7/2} [(\frac{T}{T'})^{5/2} - 1] \quad , \quad (15b)$$

and now

$$(\frac{T^*}{T})^{7/2} = \mu\alpha\xi^6 \quad (15c)$$

The salient features of (15a) are the following. First, if either $\mu \rightarrow 0$ or $\xi \rightarrow 0$, then $f \rightarrow f^*(T)$. Second, $f(T_c) = f^*(T_c)$ in accord with (5a). Third, for $T > T_c$ and $T^* > T$, the second term in (15a) is small compared to the first. It therefore follows that

$$\int_0^1 f(\mu, \xi, T)\, d\mu \simeq \frac{\ln(1+\alpha\xi^6)}{\alpha\xi^6} f^*(T) \quad , \quad (16)$$

so that there are significantly fewer suprathermal electrons propagating up the temperature gradient than in LTE.

The contribution of downward propagating electrons to the angle-averaged distribution in the velocity range $1 < \alpha\xi^6 < (T_h/T)^{7/2}$ can be found by integrating (13) over μ between the ranges $-1 \leq \mu < -(\alpha\xi^6)^{-1}$. Using (10) and (13) and defining $\mu_o(\xi,T)$ to be $(\alpha\xi^6)^{-1}$, I find that

$$J^o\ (\xi,T) \simeq \frac{1}{2} \int_{\mu_o}^{1} f(-\mu,\xi,T)\ d\mu \tag{17a}$$

$$= \frac{(7\pi)^{1/2}}{\alpha\xi^5}\ C_M(T) \int_{\mu_o^{\frac{1}{7}}}^{1} \exp\ [-\frac{7}{5}\ \xi^2 y^2 (1 - \frac{2}{7}\ y^5)]\ \frac{dy}{y^2}\ . \tag{17b}$$

Expanding the argument of the exponential about $y = \mu_o^{1/7}$ yields

$$J^o\ (\xi,T) \simeq \frac{5}{2}\sqrt{\frac{\pi}{7}}\ \frac{f^*(\bar{T}^*)}{\xi(\bar{T}^*)}\ \exp\ [-\ \frac{2}{5}\ \xi(\bar{T}^*)^2 \left\{1 - \left(\frac{T}{\bar{T}^*}\right)^{5/2}\right\} \left[1 + O(\alpha\xi^6)^{-5/7}\right]$$

$$;\ 1 < \alpha\xi^6 < \frac{T_h}{T}^{7/2} \tag{17c}$$

where f^* is a local Maxwellian, $\xi\ (\bar{T}^*) = (\frac{T}{T^*})^{1/2}\ \xi$, and $\bar{T}^* = T^*(v,\mu = -1)$; that is,

$$\frac{\bar{T}^*}{T} = (\alpha\xi^6)^{2/7} \tag{17d}$$

Note that $\bar{T}^* = \bar{T}^*\ (v)$ only. By using the relation $\alpha\xi^6|_{T=\bar{T}^*} = 1$, equation (17d) can be interpreted in a way similar to that in which equation (13) was interpreted.

On comparing (17c) with (16) it is seen that upward propagating electrons make a negligible contribution to the angle-averaged distribution for $\alpha\xi^6 >> 1$. Thus,

$$f^o(\xi,T) = \frac{5}{2}\sqrt{\frac{\pi}{7}}\ \frac{f^*(\bar{T}^*)}{\xi(T^*)}\ \exp\left[-\frac{2}{5}\ \xi(\bar{T}^*)^2\left[1-\left(\frac{T}{T^*}\right)^{5/2}\right]\right]\left[1+O\ (\alpha\xi^6)^{-5/7}\right] \tag{18a}$$

$$= \frac{5}{2}\sqrt{\frac{\pi}{7}}\ \alpha(T)^{-4/7}\ \xi(T)^{-31/7}\ C_M(T)\ \exp\left[-\frac{7}{5}\left(\frac{\xi(T)}{\alpha(T)}\right)^{2/7}\left[1-\frac{2}{7}(\alpha\xi^6)^{-5/7}\right]\right]\times$$

$$\left(1+O\ (\alpha\xi^6)^{-5/2}\right);\ \text{(18b)}$$

$$1 \leq \alpha\xi^6 \leq \left(\frac{T_h}{T}\right)^{7/2}$$

Two aspects of these formulas are worth mentioning. F--st, since both α and T are proportional to $T^{-1/2}$, while C_M is proportional to $T^{-5/2}$, it is seen that f^o depends on T only through the $(\alpha\xi^6)^{-5/7}$ term in the exponential. The spatial dependence of the tail is therefore quite weak, as seen earlier in Figure 2. Second, at low temperatures (large α), the primary velocity dependence in (18b) is through the $\xi^{-31/7}$ term; the exponential dependence is relatively weak. For example, in going from $\xi = 3$ to $\xi = 20$ at $T = 5 \times 10^4$ K ($\alpha = 9.23 \times 10^{-3}$), the exponential term decreases by a factor of 1.9×10^2, while the $\xi^{-31/7}$ term decreases by a factor of 4.5×10^3. In low temperature regions the angle-averaged distribution is therefore nearly a power law, as was also seen earlier in Figures 1 and 2.

Multiplying equation (13) by μ and integrating from $\mu = -1$ to $\mu = -(\alpha\xi^6)^{-1}$ yields

$$f^1(\xi,T) \simeq -\frac{3}{2}\int_{\mu_o}^{1} f(-\mu,\xi,T)\,d\mu \quad . \tag{19a}$$

$$= -3\,f^o\,(\xi,T) \quad ; \; 1 \le \alpha\xi^6 \le \left(\frac{T_h}{T}\right)^{7/2} \tag{19b}$$

where $f^o(\ ,T)$ is given by (18a) or (18b). Thus f^1 is also nearly a spatially independent, $v^{-31/7}$ power law in the low temperature regions of the atmosphere. This fact accounts for the slow falloff of the heat flux integrand shown in Figure 3.

3. Generalizations

The above results are easily generalized to the case in which the background temperature profile is of the form

$$T^r = T_h^{\ r} + (T_h^{\ r} - T_c^{\ r})\,\frac{z}{L} \quad ; \quad r \ge 1 \tag{20a}$$

$$n(z)T(z) = \text{constant} \tag{20b}$$

The results are:

$$\alpha_r(T) = \frac{\lambda(T_h)}{rL}\left(\frac{T}{T_h}\right)^3 \frac{T_h^{\ r} - T_c^{\ r}}{T^r} \tag{21a}$$

$$\frac{\bar{T}^*}{\bar{T}} = (\alpha_r\xi^6)^{1/r} \tag{21b}$$

$$f^o(\xi,T) = (r-1)\left(\frac{\pi}{2r}\right)^{1/2} \frac{f^*(\bar{T}^*)}{\xi(\bar{T}^*)} \exp\left[-\frac{1}{r-1}\,(\bar{T}^*)^2\left[1-\left(\frac{T}{\bar{T}^*}\right)^{r-1}\right]\right] * \tag{21c}$$

$$\left[1 + 0\,(\alpha_r\xi^6)\right]^{-\left(\frac{r-1}{r}\right)}$$

where the last equation is valid for

$$1 \leq \alpha_r \xi^6 \leq \left(\frac{T_h}{T}\right)^r \tag{21d}$$

The case $r = 3$ is of special interest, for here α_r = constant, and both f^0 and f^1 reduce to pure v^{-5} power laws.

IV. Summary

In the preceding I have shown that under inhomogeneous conditions in which only the tail of the electron velocity distribution departs significantly from a local Maxwellian distribution, as is the case in the solar transition region, a linearized BGK collision term with collision frequency proportional to v^{-3} gives results for the tail of the distribution which are in adequate agreement with results obtained from the Landau equation. Approximate analytical expressions were obtained which show that the zeroth and first angular moments of the distribution (and, undoubtedly, higher order moments) are nearly power laws over a wide velocity range in the lower temperature parts of the atmosphere.

The linear BGK model discussed here does not preserve the collisional invariants and hence is not useful for determining the kinetic temperature profile corresponding to energy transfer solely by conduction. Nevertheless, the model does unambiguously show that under appropriate conditions (say, $\alpha(T) > 10^{-3.5}$ over a substantial temperature range) the energy flux into the lower temperature regions is carried by suprathermal electrons with velocities much larger than local thermal velocities. This is in contrast to the classical prediction that the energy flux is carried by electrons with velocities two to three times local thermal velocities.

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Table I

Parameters in BGK Solutions

		$T = 5 \times 10^4$ K, $\alpha = 9.26 \times 10^{-3}$		$T = 3 \times 10^5$ K, $\alpha = 3.77 \times 10^{-3}$	
ξ	μ	$\frac{T^*(\mu,\xi)}{T}$	$\xi(T^*)$	$\frac{T^*(\mu,\xi)}{T}$	$\xi(T^*)$
3	-1.0	1.73	2.28	1.33	2.61
3	-0.5	1.42	2.51	1.09	2.87
4	-1.0	2.82	2.38	2.19	2.72
4	-0.5	2.31	2.63	1.80	2.98
5	-1.0	4.14	2.46	3.20	2.81
5	-0.5	3.40	2.71	2.63	3.08
7	-1.0	7.37	2.58	5.71	2.93
7	-0.5	6.04	2.85	4.68	3.24
10	-1.0	13.6	2.71	--	--
10	-0.5	11.2	2.99	--	--
15	-1.0	27.2	2.87	--	--
15	-0.5	22.3	3.17		

Figure Captions

Figure 1 Dimensionless, angle-averaged distibution function versus $\xi = v/v_{th}$. Solid lines, Landau equation results. Broken lines, BGK results. Dashed line, a local Maxwellian distribution.

Figure 2 Dimensional, angle-averaged distribution function f^{o} versus velocity measured in units of the thermal velocity at the upper boundary. Solid lines are BGK results; dashed lines are local Maxwellians.

Figure 3 Absolute value of the heat flux integrand versus velocity mesured in units of thermal velocity at the upper boundary. Solid lines, full BGK result. Dashed line, classical transport theory prediction for BGK equation.

Figure 4 Heat flux calculated from linear BGK equation (normalized to its classical value) versus temperature. Results for several pressures are shown.

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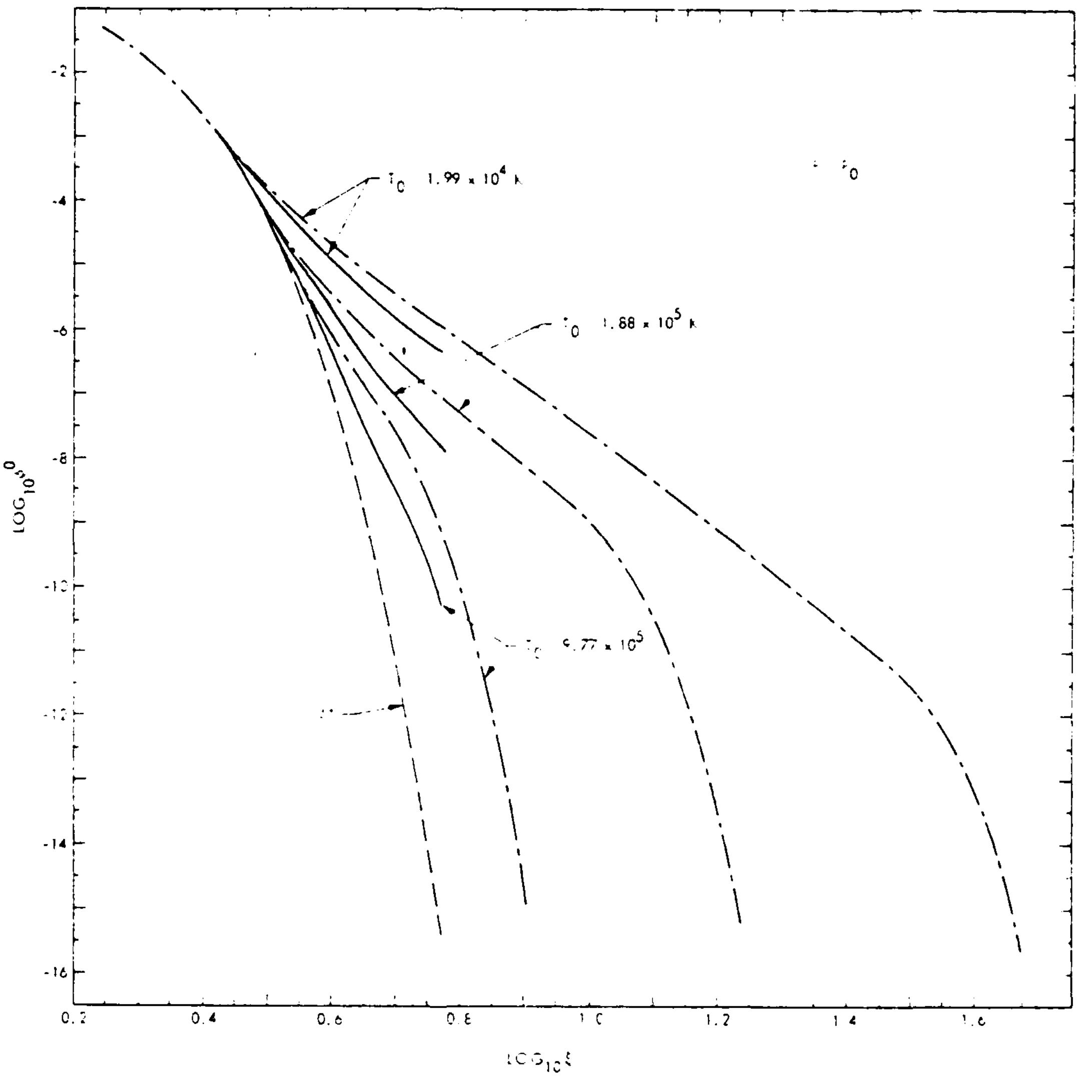
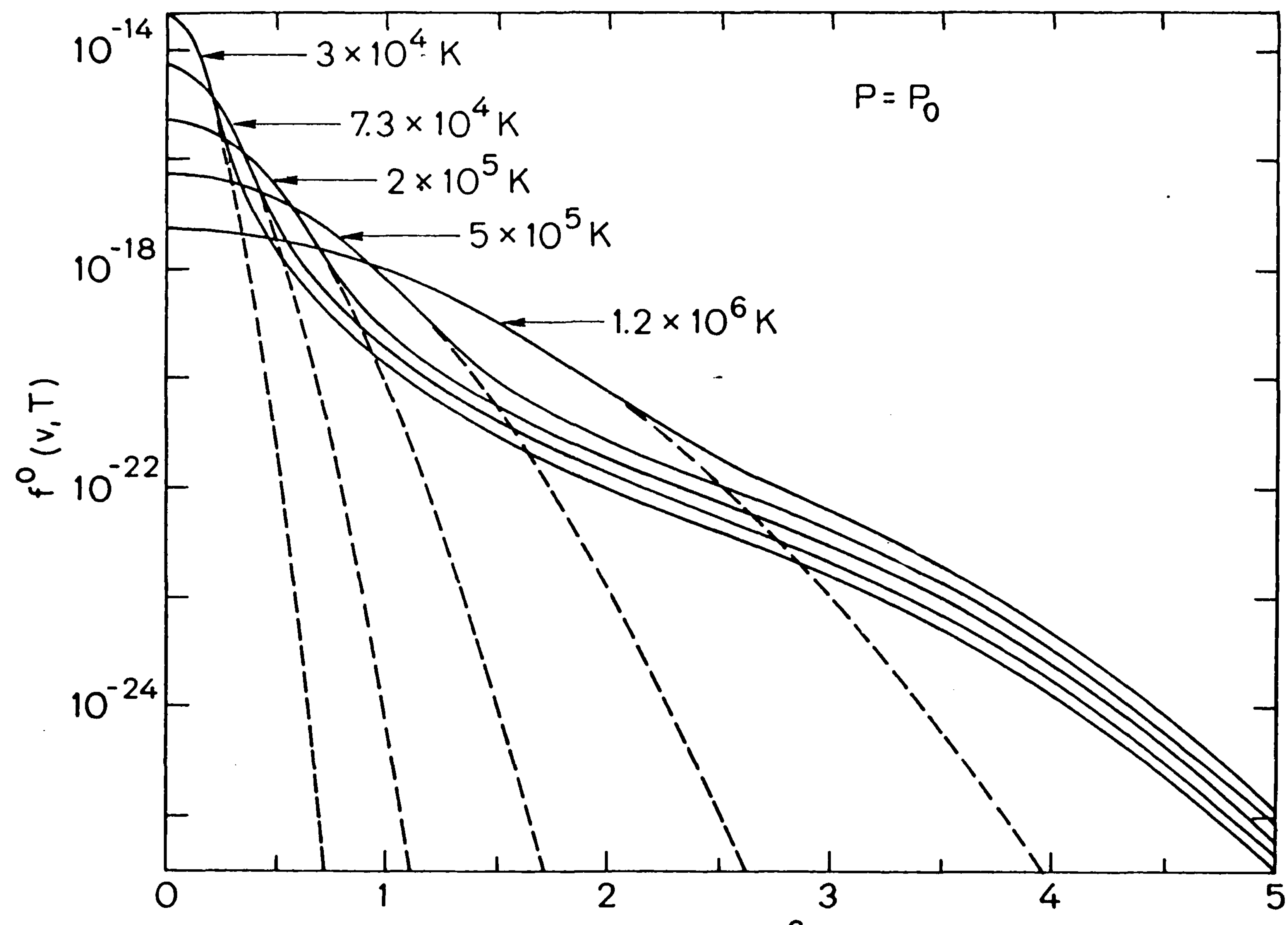


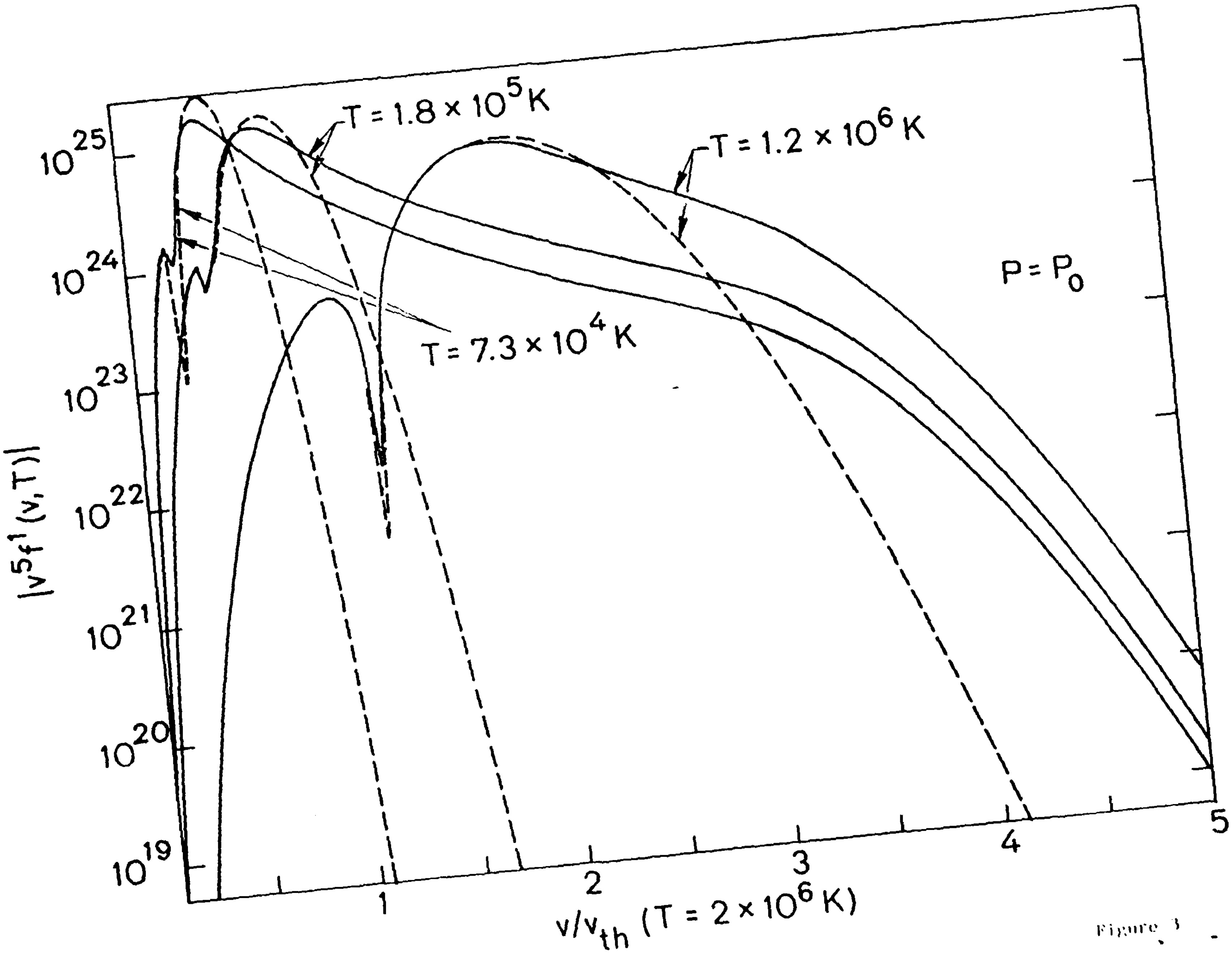
T_0 1.99 x 10^4 K
T_0 1.88 x 10^5 K
T_0 9.77 x 10^5
-2
-4
-6
-8
-10
-12
-14
-16
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

Figure 1



10^{-14}
10^{-18}
10^{-22}
10^{-24}
$f^0(v,T)$
3×10^4 K
7.3×10^4 K
2×10^5 K
5×10^5 K
1.2×10^6 K
$P = P_0$
0
1
2
3
4
5
$v/v_{th}\,(2 \times 10^6\,\mathrm{K})$

Figure 2



T = 1.8 × 10⁵ K
T = 1.2 × 10⁶ K
T = 7.3 × 10⁴ K
P = P0
|v5f1(v,T)|
10^25
10^24
10^23
10^22
10^21
10^20
10^19
1
2
3
4
5
v/vth (T = 2 × 10⁶ K)

Figure 3

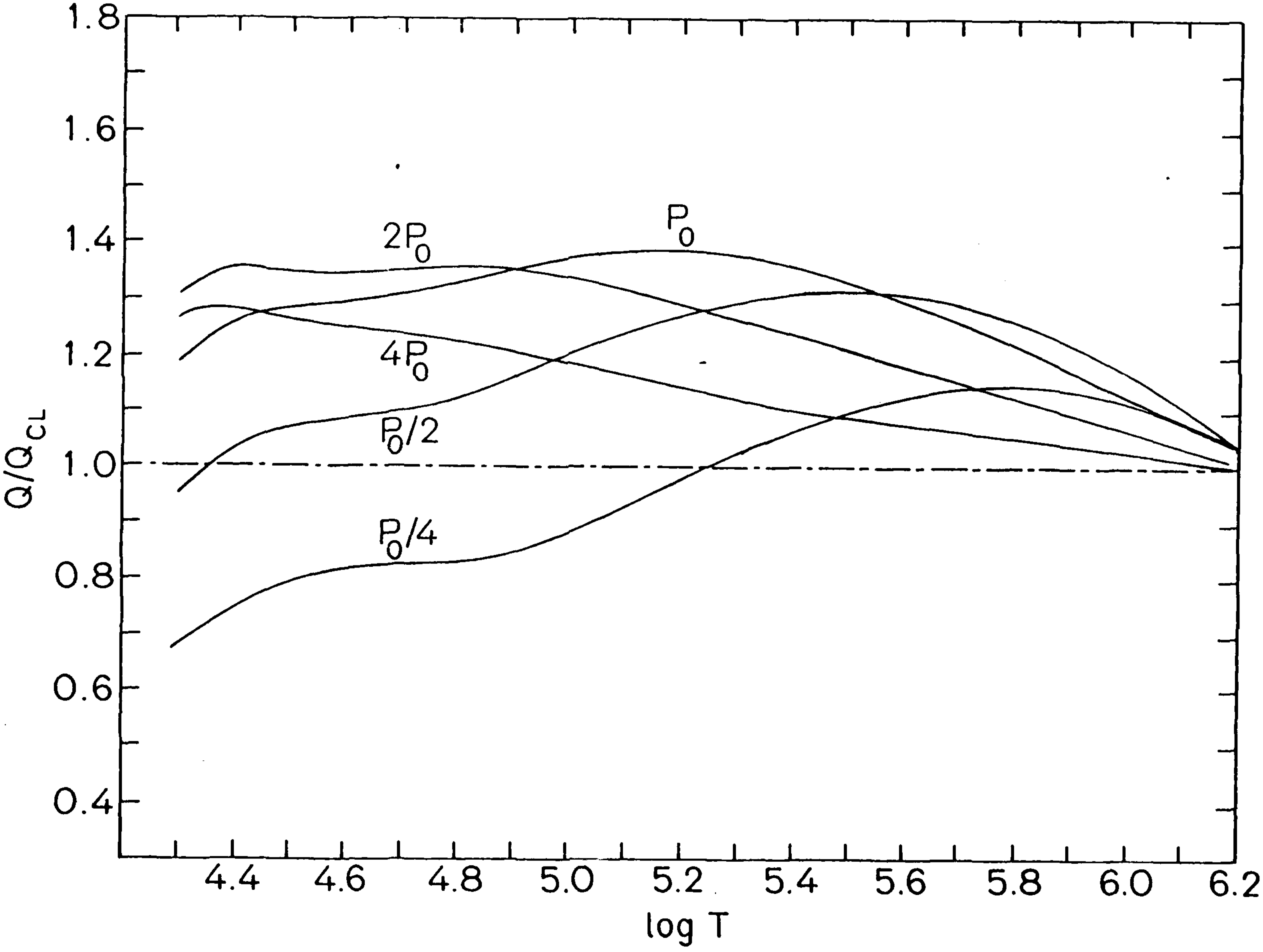

1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
Q/Q_CL
2P_0
P_0
4P_0
P_0/2
P_0/4
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
6.2
log T

Figure 4